

CORRECTION TO THE DYNAMIC TENSILE STRENGTH OF ICE AND ICE-SILICATE MIXTURES (LANGE & AHRENS 1983). Sarah T. Stewart, Thomas J. Ahrens, *Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, CA 91125, USA (sstewart@gps.caltech.edu)*, Manfred A. Lange, *Institute for Geophysics, University of Muenster, Germany.*

Lange & Ahrens ([1], hereafter LA83) conducted dynamic tensile strength experiments in one dimensional strain on ice and ice-silicate mixtures to determine the strain rate dependence and derive parameters useful for continuum fracturing models. They combined their data, at strain rates of $2 \times 10^4 \text{ s}^{-1}$, with similar data from Hawkes & Mellor [2] at strain rates of 1 s^{-1} to calculate the Weibull parameters for each mixture. To date, no new experimental data have been published regarding the dynamic tensile strength or strain rate dependence on strength in ice and ice-silicate mixtures. Unfortunately, LA83 contained an error in the calculation of the Weibull parameters which we correct here.

Sand Content		Density	V_p	C_g	K
wt %	vol %	g/cm^3	km/s	km/s	GPa
0	0	0.917	3.83	1.53	9.47
5	1.8	0.948	3.51	1.41	6.50
30	12.9	1.141	3.65	1.46	8.44

Table 1: Sample properties

The properties of the ice and ice mixtures are described in Table 1. LA83 performed experiments on three different mixtures and found that, at strain rates of $2 \times 10^4 \text{ s}^{-1}$, pure ice has a tensile strength of $\sim 17 \text{ MPa}$ and ice-silicate mixtures with 5 and 20 wt% sand have tensile strengths of ~ 20 and 22 MPa respectively.

LA83 assumed that the activation of cracks could be described by a Weibull distribution, where the number of cracks activated at or below a tensile strain level ϵ is

$$n = k\epsilon^m \quad (1)$$

where k and m are material parameters.

Following the derivation of Grady & Kipp [3], the fracture strength of a material is given by

$$\sigma_M = K(m+3)(m+4)^{-(m+4)/(m+3)} \alpha^{-1/(m+3)} \dot{\epsilon}^{3/(m+3)} \quad (2)$$

where K is the bulk modulus and

$$\alpha = \frac{8\pi C_g^3 k}{(m+1)(m+2)(m+3)} \quad (3)$$

where C_g is the velocity of crack growth.

The assumptions and approximations made by LA83 are as follows:

- 1 The tensile strength of each mixture was estimated by visual inspection of recovered samples within a relatively narrow transition between spallation and fragmentation, see LA83's Fig. 3.

- 2 They calculated the bulk modulus based on their measurements of the P wave velocities, V_p , in each mixture and applied the often used Lamé approximation that the shear wave velocity $= \sqrt{3}V_p$.

- 3 They assume that $V_p/3 \leq C_g \leq 2V_p/5$ and use $C_g = 0.4V_p$ in their calculations (Table 1).

- 4 They estimate the tensile strength of the ice-silicate mixtures at a strain rate of 1 s^{-1} by scaling the tensile strength of ice ($\sim 1.6 \text{ MPa}$ [2]) by the ratios found at a strain rate of 2×10^4 , so the values of k and m found for these mixtures can only be considered estimates.

- 5 LA83 estimated L_M , the typical fragment size, from visual examination. This value was used to constrain m and k , but it is not clear how they did this.

There is an error in LA83's calculation of m and k . It is not clear where the error occurred, but their Fig. 7 shows a fit to a tensile strength of 17 GPa , when the data are actually in MPa . Using the values of C_g , K , and σ_M (observed) from LA83, the Weibull parameters have been recalculated and the revised values are shown in Table 2. We did not use L_M , t_M the time to reach maximum stress, or t_f the time to reach tensile failure in the calculation of m and k as these values are too poorly constrained.

Sand wt %	LA83		Corrected Value	
	m	k	m	k
0	8.7	0.32×10^{45}	9.57	1.28×10^{38}
5	9.4	0.56×10^{44}	9.56	1.79×10^{35}
30	9.4	0.56×10^{45}	9.57	1.34×10^{36}

Table 2: Revised calculation of the Weibull parameters

The values of L_M , t_M , and t_f are calculated with the revised m and k (following [3]). The values of m and k were fit to σ_M , so the observed and theoretical values are identical. The recalculated values are shown in Table 3.

The values for L_M , t_M , and t_f are similar to LA83 because they have a stronger dependence on m than on k . With the new value of $m = 9.57$, the tensile strength of ice is dependent on the strain rate by

$$\sigma_M \propto \dot{\epsilon}_0^{3/(m+3)} = \dot{\epsilon}_0^{0.239} \quad (4)$$

where the strain rate is assumed to accelerate quickly to a constant value, $\dot{\epsilon}_0$. Granitic rock and concrete demonstrate a similar value of $3/(m+3) = 1/3$ for the dependence on strain rate [3].

More data are required to provide better constraints and error estimates on the Weibull parameters. We are conducting

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new dynamic tensile strength experiments on solid ice and have begun a study of porous ice to refine these parameters and to model the effect of porosity on impact phenomenon.

References: [1] M.A. Lange and T.J. Ahrens, *J. Geophysical Res.* **88**, 1197-1208, 1983. [2] I. Hawkes and M. Mellor, *J. Glaciol.* **11**, 103-131, 1972. [3] D.E. Grady and M.E. Kipp, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.* **17**, 147-157, 1980.

Sand wt %	$\dot{\epsilon}$ s^{-1}	σ_M MPa	
		Observed	Theoretical
0	1	1.60	1.60
	2×10^4	17.0	17.0
5	1	1.88	1.88
	2×10^4	20.0	20.0
30	1	2.07	2.07
	2×10^4	22.0	22.0
		L_M mm	
		Observed	Theoretical
0	1		178
	2×10^4	0.1-0.5	0.0946
5	1		281
	2×10^4	0.1-0.5	0.150
30	1		247
	2×10^4	0.1-0.5	0.131
		t_M μs	
		Observed	Theoretical
0	1		182
	2×10^4	$\leq 0.3 - 0.75$	0.0970
5	1		312
	2×10^4	$\leq 0.3 - 0.75$	0.166
30	1		265
	2×10^4	$\leq 0.3 - 0.75$	0.141
		t_f μs	
		Observed	Theoretical
0	1		224
	2×10^4	$\leq 0.3 - 0.75$	0.120
5	1		384
	2×10^4	$\leq 0.3 - 0.75$	0.204
30	1		325
	2×10^4	$\leq 0.3 - 0.75$	0.173

Table 3: New calculations of σ_M , L_M , t_M , t_f with the revised Weibull parameters, k and m .